Stress-Strain Relationship for Steel under Uniaxial Cyclic Loadings

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Abstract: The mechanical properties of steel in the inelastic range can generally be described by mathematical relationships. Many such constitutive relationships have been validated by static or uniaxial cyclic loading tests. Very few models have been substantiated by test results under complex loading conditions. For that reason, the implementation of such models in general purpose structural analysis programs for steel structures under seismic actions, is in some cases complex and in others impossible. This paper is concerned with a uniaxial non-linear model for structural steel under complex loading condition and with damage accumulation. The Giuffré, Menegoto and Pinto model was taken as a basis for the development of this model. The accuracy of the proposed numerical model was drawn with uniaxial cyclic experiments. Some numerical simulations are presented in order to illustrate the capabilities of the model for use as a stress-strain relationship for steel under uniaxial complex loading conditions up to the complete failure of the material.

Key words: Stress-strain relationship, damage-index, cyclic behaviour, steel, numerical modelling.

NOTATION

A_1 constant
A_2 constant
b parameter
E modulus of elasticity
F force
f function for the decrease of the stress
I_d damage index
I_d^T total damage index
I_d^c damage index related with the complete semi-cycles
k_σ parameter related with the amplitude of the stress
k_ε parameter related with the amplitude of the strain
K parameter used in the damage index
m parameter used in the damage index
N_j number of complete semi-cycles to failure
R parameter that takes into account the Bauschinger effect
R_0 constant
SGN function that takes into account the sign of the argument.
ε_s^* relative strain
ε_s strain
ε_{sr} strain at the last loading inversion
ε_{sy} yield strain
ξ_{max} maximum plastic excursion
σ_s^* relative stress
σ_s stress and stress with damage accumulation
$\hat{\sigma}_s$ stress without damage accumulation
σ_{sr} stress at the last loading inversion
$\hat{\sigma}_{sr}$ stress at the last inversion without damage accumulation
σ_{sy} yield stress
$\frac{d\sigma}{d\varepsilon}$ tangent stiffness
δ displacement
ΔI_d^c damage index related to the current semi-cycle.

1. INTRODUCTION

Non-linear analysis of steel structures, such as those appropriate for earthquake ground motions, often require consideration of the inelastic behaviour of the material. In recent years, the finite element method has been widely used for non-linear structural analysis. However, despite the high level of sophistication in many finite element numerical schemes, the accuracy of the results depends inevitably on the modelling of the material used.

In recent decades, several numerical models have been developed to describe material behaviour in the plastic range. Among them should be pointed out the models developed by Ramberg and Osgood (1943), Prager (1956), Giuffrè and Pinto (1970), Aktan et al. (1973), Menegotto and Pinto (1973), Kato et al. (1973), Ma et al. (1976), Petersson and Popov (1977), Drucker and Palgen (1981), Chang and Lee (1990), Castiglioni (1990), Monti and Nuti (1990), Nelson and Dorfmann (1995), Balan et al. (1998), Rodriguez et al. (1999) and Rodzik (1999).

In this context there are two approaches: one using the constitutive relation in the form $\varepsilon = f(\sigma)$ as for the model proposed by Ramberg and Osgood (1943); the other, in the form $\sigma = f(\varepsilon)$ as for the model developed by Menegotto and Pinto (1973).

Some of the above models often fail to agree well with experimental data, especially for complex cyclic loadings involving partial unloading and damage accumulation. The model proposed by Menegotto and Pinto (1973), it should be noted, offers significant computational advantages when the finite-element formulation is based on geometric (kinematic) approximations.

In this paper an improvement to the model proposed by Giuffré and Pinto (1970) and Menegoto and Pinto (1974) is presented. The main characteristics are: (1) cyclic loadings with partial unloading are considered; (2) damage accumulation for the stress-strain relationship of the steel is included. The model is based on an uniaxial cyclic stress-strain relation and takes into account the degradation of the strength properties of the material with accumulation of plastic strains. The accuracy and validity of the proposed numerical model is checked with experimental tests performed on steel specimens under cyclic uniaxial loadings.

2. THE GUIFFRÉ, MENEGOTO & PINTO MODEL'S

The model proposed by Guiffré and Pinto (1970) for the stress-strain relationship of steel under uniaxial cyclic loading can be represented by the following Eqn (1):

$$\sigma_s^* = \frac{\varepsilon_s^*}{\sqrt[R]{1+|\varepsilon_s^*|^R}} \tag{1}$$

In this equation ε_s^* and σ_s^* represent respectively the relative strain and relative stress. For the 1st half-cycle ε_s^* and σ_s^* can be obtained by the following Eqns (2):

$$\varepsilon_s^* = \frac{\varepsilon_s}{\varepsilon_{sy}} \qquad \sigma_s^* = \frac{\sigma_s}{\sigma_{sy}} \tag{2}$$

For the following half-cycles these parameters can be obtained by Eqns (3):

$$\varepsilon_s^* = \frac{\varepsilon_s - \varepsilon_{sr}}{2\varepsilon_{sy}} \qquad \sigma_s^* = \frac{\sigma_s - \sigma_{sr}}{2\sigma_{sy}} \tag{3}$$

In these equations the variables have the following meaning:

ε_s strain
ε_{sr} strain at the last loading inversion
ε_{sy} yield strain
σ_s stress
σ_{sr} stress at the last loading inversion
σ_{sy} yield stress

In this model the stress-strain relationship is elasto-plastic and has a transition curve that is a function of the parameter R. The larger the value of the parameter R the more similar the curve is to elastic perfectly plastic behaviour. The model proposed by Guiffré and Pinto (1970) has two asymptotic lines that are respectively $\sigma = \pm\sigma_{sy}$. Unloadings are obtained with an initial stiffness equal to the modulus of elasticity, Eqn (4):

$$E = \frac{\sigma_{sy}}{\varepsilon_{sy}} \tag{4}$$

In this model the parameter R takes into account the Bauschinger effect and defines the shape of the transition curve between the elastic and plastic zones and is a function of the maximum plastic excursion (ξ_{max}) obtained until the step. Thus, the parameter R depends on the loading history and can be assessed by Eqn (5):

$$R = R_0 - \frac{A_1 \xi_{max}}{A_2 + \xi_{max}} \tag{5}$$

In this equation R_0, A_1 and A_2 are constants. The maximum plastic excursion can be obtained by the following equation (6):

$$\xi_{max} = \max_i \left\{ \left| \varepsilon_{sr}^{(i)} - \varepsilon_{sr}^{(i-1)} + \frac{\sigma_{sr}^{(i-1)} - \sigma_{sr}^{(i)}}{E} \right| \right\} \quad (6)$$

where ($\varepsilon_{sr}^{(i)}$, $\sigma_{sr}^{(i)}$) represent load inversion i.

Hardening was later included in the model (Menegotto and Pinto (1973)):

$$\sigma_s^* = b\,\varepsilon_s^* + \frac{(1-b)\varepsilon_s^*}{\sqrt[R]{1+|\varepsilon_s^*|^R}} \quad (7)$$

where b is the ratio between the kinematic stiffness hardening and the modulus of elasticity:

$$b = \frac{E_{kin}}{E} \quad (8)$$

This stress-strain relationship has as asymptotes the lines with slope E_{kin} and which include points ($\pm\varepsilon_{sy}$, $\pm\sigma_{sy}$). The plastic excursion (Fig. 1) can be obtained from Eqn (9):

$$\xi_{\max} = \max_i \left\{ \frac{1}{1-b} \times \left| \varepsilon_{sr}^{(i)} - \varepsilon_{sr}^{(i-1)} + \frac{\sigma_{sr}^{(i-1)} - \sigma_{sr}^{(i)}}{E} \right| \right\} \quad (9)$$

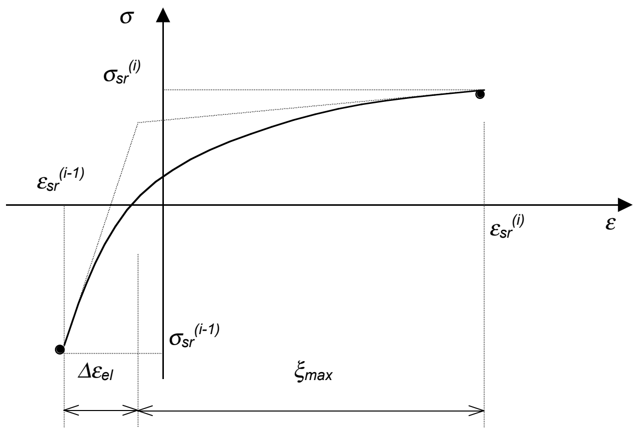

Figure 1. Plastic excursion

The model developed by Menegotto and Pinto (1973) has some limitations when used in seismic analysis, due to the fact that partial unloading is not considered and equality is imposed between the positive and negative yield stresses. However, its formulation seems to be suitable for including these types of loading and material characteristics.

The improvements proposed for the Menegotto and Pinto (1973) model, in order to make it more general are presented below.

3. MODEL FOR $\sigma-\varepsilon$ RELATIONSHIP WITH UNLOADING

The model developed is based on the Menegotto and Pinto (1973) model, reformulated to take into account the effects of unloading. In this reformulation new expressions are proposed to assess the relationship between the relative stress (σ_s^*) and the relative strain (ε_s^*). The relative stress and relative strain can be obtained (Brito 1999) from the following Eqns (10):

$$\sigma_s^* = \frac{\hat{\sigma}_s - \hat{\sigma}_{sr}}{k_\sigma \sigma_{sy}} \qquad \varepsilon_s^* = \frac{\varepsilon_s - \varepsilon_{sr}}{k_\varepsilon \sigma_{sy}} \quad (10)$$

where

$\hat{\sigma}_s$ stress without damage accumulation
$\hat{\sigma}_{sr}$ stress at the last inversion without damage accumulation

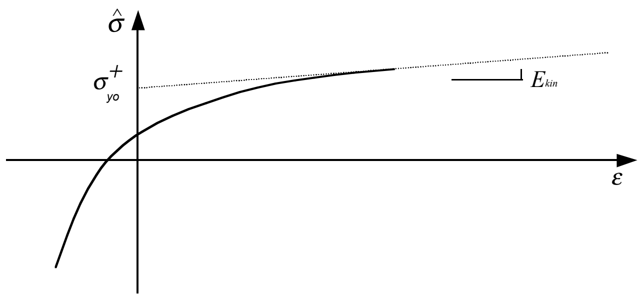

Figure 2. Asymptotic behaviour of the stress-strain relationship

The parameters k_σ and k_ε related with the amplitude of the stress and strain are obtained by imposing the stress-strain curve to be asymptotic to the kinematic hardening curve, as is shown in Fig. 2:

$$\hat{\sigma}_s \to \sigma_{yo}^+ + E_{kin}\varepsilon_s \quad (11)$$

From the equation proposed by Menegotto and Pinto (1973) it is possible to obtain the following Eqn (12):

$$\sigma_s^* \to b\varepsilon_s^* + (1-b) \quad (12)$$

Replacing the relative stress and the relative strain (10) in Eqn (12), the latter can be re-written as:

Table 1. Equations for the stress-strain relationship considering the partial unloading

Eqn	ε_s increasing	ε_s decreasing
10	$\sigma_s^* = \dfrac{\hat{\sigma}_s - \hat{\sigma}_{sr}}{k_\sigma \sigma_{sy}} \quad \varepsilon_s^* = \dfrac{\varepsilon_s - \varepsilon_{sr}}{k_\varepsilon \sigma_{sy}}$	$\sigma_s^* = \dfrac{\hat{\sigma}_s - \hat{\sigma}_{sr}}{k_\sigma \sigma_{sy}} \quad \varepsilon_s^* = \dfrac{\varepsilon_s - \varepsilon_{sr}}{k_\varepsilon \sigma_{sy}}$
11	$\hat{\sigma}_s \to \sigma_{yo}^+ + E_{kin}\varepsilon_s$	$\hat{\sigma}_s \to \sigma_{yo}^- + E_{kin}\varepsilon_s$
12	$\sigma_s^* \to b\varepsilon_s^* + (1 - b)$	$\sigma_s^* \to b\varepsilon_s^* - (1 - b)$
13	$\hat{\sigma}_s \to \dfrac{k_\sigma}{k_\varepsilon} E_{kin}\varepsilon_s + \left[k_\sigma \sigma_{sy}(1 - b) - \dfrac{k_\sigma}{k_\varepsilon} E_{kin}\varepsilon_{sr} + \hat{\sigma}_{sr} \right]$	$\hat{\sigma}_s \to \dfrac{k_\sigma}{k_\varepsilon} E_{kin}\varepsilon_s + \left[k_\sigma \sigma_{sy}(1 - b) - \dfrac{k_\sigma}{k_\varepsilon} E_{kin}\varepsilon_{sr} + \hat{\sigma}_{sr} \right]$
14	$\dfrac{k_\sigma}{k_\varepsilon} = 1$	$\dfrac{k_\sigma}{k_\varepsilon} = 1$
15	$k_\sigma = 1 - \dfrac{\hat{\sigma}_{sr}}{(1 - b)\sigma_{sy}} + \dfrac{b}{1 - b}\dfrac{\varepsilon_{sr}}{\varepsilon_{sy}}$	$k_\sigma = 1 + \dfrac{\hat{\sigma}_{sr}}{(1 - b)\sigma_{sy}} - \dfrac{b}{1 - b}\dfrac{\varepsilon_{sr}}{\varepsilon_{sy}}$

$$\hat{\sigma}_s \to \left[\frac{k_s \sigma_{sy}}{k_\varepsilon \varepsilon_{sy}} b \right] \varepsilon_s + \left[k_s \sigma_{sy}(1 - b) - \frac{k_\sigma \sigma_{sy}}{k_\varepsilon \varepsilon_{sy}} b\varepsilon_{sr} + \hat{\sigma}_{sr} \right] \quad (13)$$

Comparing (11) with (13) the following equations can be obtained (14-15):

$$\frac{k_\sigma}{k_\varepsilon} = \frac{E_{kin}}{Eb} = 1 \quad (14)$$

$$k_\sigma = \frac{\sigma_{yo}^+ - \hat{\sigma}_{sr} + E_{kin}\varepsilon_{sr}}{(1 - b)\sigma_{sy}} = 1 - \frac{\hat{\sigma}_{sr}}{(1 - b)\sigma_{sy}} + \frac{b}{1 - b}\frac{\varepsilon_{sr}}{\varepsilon_{sy}} \quad (15)$$

The implementation of this stress-strain relationship not only needs the assessment of the stress as function of the strain but also the evaluation of the tangent stiffness $d\sigma/d\varepsilon$.

The main expressions to assess this stress-strain relationship, which take into account the partial unloading, are (Brito 1999):

$$\frac{d\hat{\sigma}_s}{d\varepsilon_s} = \frac{d\hat{\sigma}_s}{d\sigma_s^*} \times \frac{d\sigma_s^*}{d\varepsilon_s^*} \times \frac{d\varepsilon_s^*}{d\varepsilon_s} = \frac{\dfrac{d\sigma_s^*}{d\varepsilon_s^*}}{\dfrac{d\sigma_s^*}{d\hat{\sigma}_s}} \times \frac{d\varepsilon_s^*}{d\varepsilon_s} \quad (16)$$

$$\frac{d\sigma_s^*}{d\hat{\sigma}_s} = \frac{1}{k_\sigma \sigma_{sy}} \quad (17)$$

$$\frac{d\varepsilon_s^*}{d\varepsilon_s} = \frac{1}{k_\varepsilon \varepsilon_{sy}} \quad (18)$$

$$\frac{d\sigma_s^*}{d\varepsilon_s^*} = b + (1 - b)\frac{1}{\left[1 + |\varepsilon_s^*|^R\right]^{1+1/R}} \qquad (19)$$

$$\frac{d\hat{\sigma}_s}{d\varepsilon_s} = E \times \left[b + (1 - b)\frac{1}{\left[1 + |\varepsilon_s^*|^R\right]^{1+1/R}}\right] \qquad (20)$$

The previous equations were deduced assuming an increasing load. For decreasing load the deduction is similar. In Table 1 all the equations for increasing and decreasing strains are written.

4. MODEL FOR σ–ε RELATIONSHIP WITH DAMAGE ACCUMULATION

To take into account damage accumulation due to cyclic loading it is necessary to include into the stress-strain relationship a damage index that should consider the deterioration in the previous complete semi-cycles and the deterioration in the current semi-cycle. A possible equation for the total damage index, Eqn (21) (Brito 1999) may be written as follows:

$$I_d^T = I_d^c + \Delta I_d^c \qquad (21)$$

where

- I_d^T total damage index;
- I_d^c damage index related with the complete semi-cycles;
- ΔI_d^c damage index related to the current semi-cycle.

The damage index related with the complete semi-cycles is constant during each semi-cycle while the damage index related to the current semi-cycle is a function of the current strain and can be assessed through the following Eqn (22) (Brito 1999):

$$\Delta I_d^c = \frac{1}{N_j} = \frac{\left(\frac{|\varepsilon_s - \varepsilon_{sr}|}{\varepsilon_y}\right)^m}{K} \qquad (22)$$

In this equation N_j is the number of complete semi-cycles to failure and m and K are parameters dependent on the typology and the mechanical properties of the component under consideration and may be obtained by a statistical evaluation, Krawinkler *et al.* (1983). In a *Log-Log* domain Eqn (22) represents a straight line with a slope equal to *-1/m* called the fatigue resistance line, which identifies the safe and unsafe regions.

Thus the value of stress taking into account the damage accumulation can be obtained from the following Eqn (23):

$$\sigma_s = \hat{\sigma}_s \times (1 - f(I_d)) \qquad (23)$$

where:

- σ_s stress with damage accumulation;
- $\hat{\sigma}_s$ stress without damage accumulation;
- f function for the decrease of the stress;
- I_d damage index.

The function for the decrease of the stress *(f)* can be obtained through experimental tests (Proença 1996) and is shown in Fig. 3.

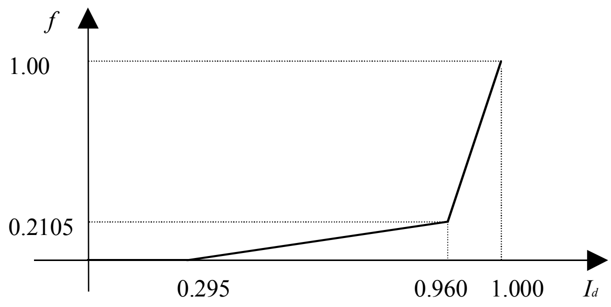

Figure 3. Function for the decrease of the stress

Analytically the function *f* can be represented by the following Eqns (24):

$$f(I_d) = \begin{cases} 0 & \Leftarrow \quad I_d \le 0.295 \\ 0.3168 I_d - 0.0936 & \Leftarrow \quad I_d \in \left]0.295;0.960\right] \\ 19.7332 I_d - 18.7332 & \Leftarrow \quad I_d > 0.960 \end{cases} \qquad (24)$$

The assessment of the stress is carried out in two steps: a first step where the stress is evaluated without consideration of the damage accumulation ($\hat{\sigma}_s$) and a second one where the stress is corrected to take into account the damage in the current semi-cycle.

In this methodology the correction of stress is made at the end of the current semi-cycle. The new equation for tangent stiffness can be obtained from the following Eqn (25):

$$\frac{d\sigma_s}{d\varepsilon_s} = \frac{d\hat{\sigma}_s}{d\varepsilon_s} \times [1 - f(I_d)] - \frac{df}{d\varepsilon_s}\hat{\sigma}_s \qquad (25)$$

To assess the value of Eqn (25) $df/d\varepsilon_s$ may be obtained from Eqn (26):

$$\frac{df}{d\varepsilon_s} = \frac{df}{dI_d^T} \times \frac{dI_d^T}{d\varepsilon_s} = \frac{df}{dI_d^T} \times \frac{d\Delta I_d^p}{d\varepsilon_s} \qquad (26)$$

with

$$\frac{df}{dI_d^T} = \begin{cases} 0 & \Leftarrow \quad I_d^T \leq 0.295 \\ 0.3168 & \Leftarrow \quad I_d^T \in [0.295;0.960[\\ 19.7332 & \Leftarrow \quad I_d^T > 0.960 \end{cases} \qquad (27)$$

and

$$\frac{d\Delta I_d^p}{d\varepsilon_s} = \frac{m}{K\,\varepsilon_y}\left(\frac{|\varepsilon_s - \varepsilon_{sr}|}{K\,\varepsilon_y}\right)^{m-1} \times SGN\,(\varepsilon_s - \varepsilon_{sr}) \qquad (28)$$

The *SGN* function takes the sign of the argument into account.

5. ACCURACY OF THE PROPOSED σ–ε MODEL

To check the accuracy and the validity of the proposed model for the stress-strain relationship of the steel with due consideration of partial unloading and damage accumulation, several experimental tests were performed at the Laboratory for Structures and Strength of Materials of the Instituto Superior Técnico, Lisbon. They consisted of uniaxial cyclic tests on S235 steel specimens (yield strength equal to 235 MPa) with $20 \times 20 \times 160$ mm^3 as general dimensions. Different types of cyclic loading were used including both increasing and constant amplitude tests. Although the slenderness of specimens is not very high some of them exhibited buckling induced initial onset of the failure. Fig. 4 presents an experimental result, which is compared with a numerical simulation, Fig. 5.

Comparison between experimental data and numerical simulation in general shows good agreement although these are quite poor correlation in the transitional range up to a strain of about 25×10^{-3}. The numerical model is dependent on the parameter *R* (Eqn (5)) that takes into account the Bauschinger effect and the shape of the transition curve between the elastic and the plastic range. The larger the value of the parameter *R* the more similar the curve is to that of elastic perfectly plastic behaviour. The constants A_1, A_2 and R_0 used in this numerical simulation were obtained from experimental tests performed at the Instituto Superior Técnico through the least square method and were: $A_1 = 18.25$, $A_2 = 3.917 \times 10^{-3}$ and $R_0 = 20$. The quite poor correlation for the transition curve in the range 0 to 25 may be attributed to the type of steel. According to Proença (1996) the parameter *R* for S235 steels (yield strength equal to 235 MPa) exhibit a poorer correlation for the transition curve that with high steels such as S355 (yield strength equal to 355 MPa). However, the proposed numerical model is able to simulate the cyclic behaviour, the Bauschinger effect and the damage accumulation until failure. Concerning the strain energy stored until the failure it was verified for this case that the error obtained is less than +8% showing the ability of the model to be used in seismic simulations.

In Figs 6 to 9 some numerical simulations are presented to show the capabilities of the model for simulating the behaviour of the steel under different loading conditions. Fig. 6 represents the numerical simulation of a specimen under uniaxial monotonic

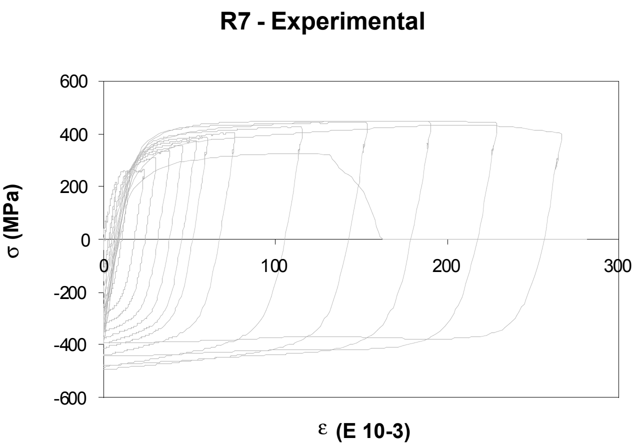

Figure 4. Uniaxial tension test of a steel specimen (Adapted from Proença (1996))

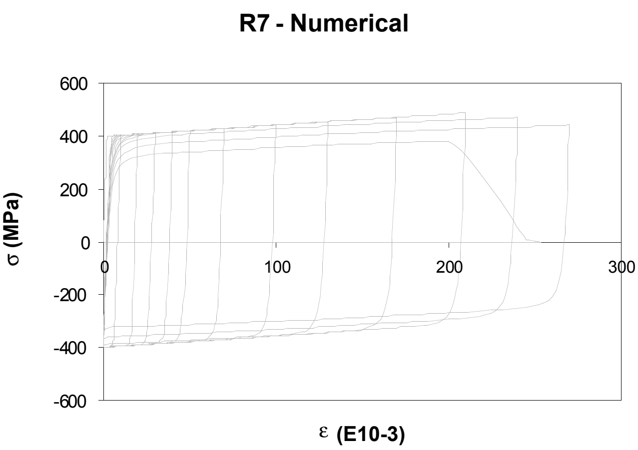

Figure 5. Numerical simulation of the uniaxial tension test

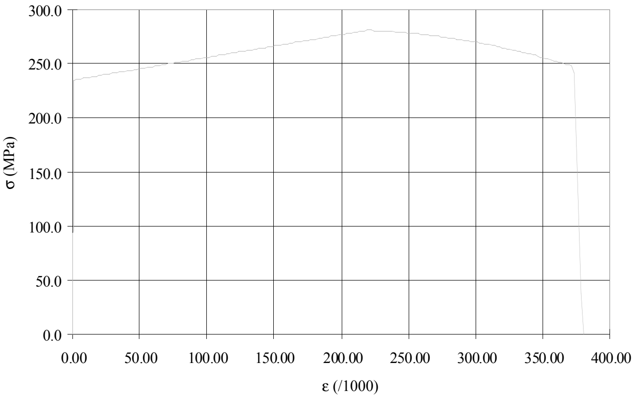

Figure 6. $\sigma - \varepsilon$ curve for a specimen under monotonic amplitude displacement

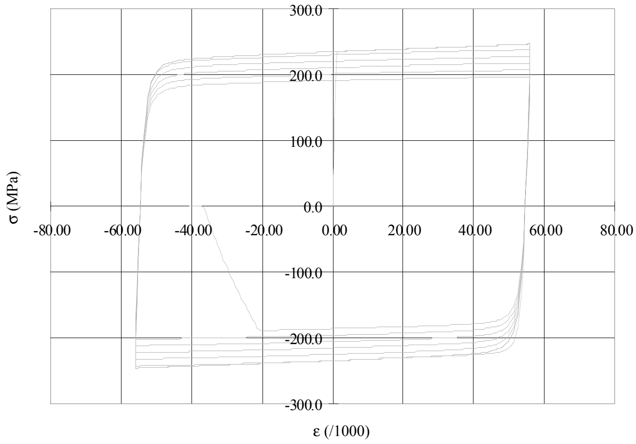

Figure 7. $\sigma - \varepsilon$ curve for a specimen under cyclic constant amplitude displacement

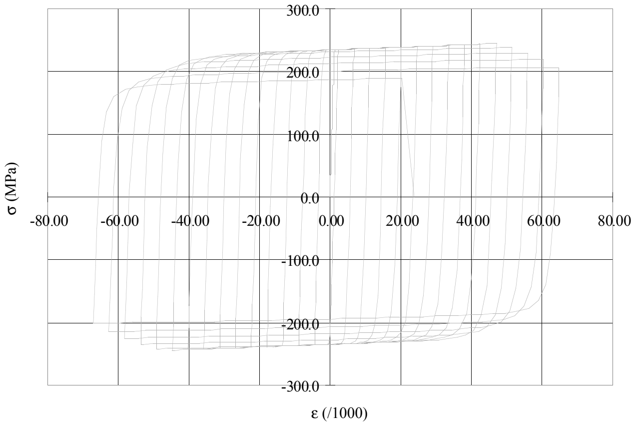

Figure 8. $\sigma - \varepsilon$ curve for a specimen under cyclic increasing amplitude displacement

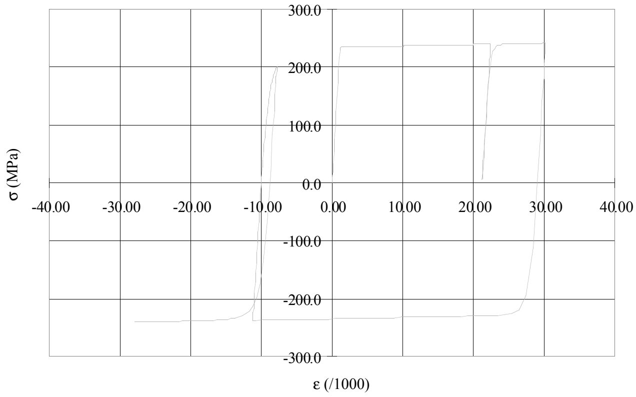

Figure 9. $\sigma - \varepsilon$ curve for a specimen under cyclic random amplitude displacement

displacement, while Fig. 7 shows the stress-strain diagram of a specimen under cyclic constant amplitude displacement. In the case of Fig. 8 a cyclic increasing amplitude loading history is considered while in Fig. 9 cyclic random amplitude simulated displacement results are shown.

The results of these four simulations provide evidence that partial unloadings and loss of stress due to damage accumulation are able to be reproduced by the proposed model allowing the conclusion that the model is suitable for the numerical simulation of steel structures under seismic actions.

6. NUMERICAL SIMULATION OF THE HYSTERETIC BEHAVIOUR OF A BEAM-TO-COLUMN CONNECTION

The stress-strain relationship presented in this paper was used in a finite element program in order to investigate its suitability in the simulation of the hysteretic behaviour of a beam-to-column connection.

The connection experimentally and numerically studied was a typical beam-to-column connection with top and seat web angle as shown in Fig. 10. The cross sections of the beam and the column were respectively a IPE300 and a HEB200. Angles L120 × 120 × 10 and two rows of preloaded M16 grade 8.8 bolts were used to connect the column to the beam. All structural elements were simulated by finite elements of beam with geometric and mechanical properties analogous to the real elements.

The accuracy and capability of the proposed model for use as the stress-strain relationship in numerical simulations of the hysteretic behaviour of steel structural elements, such as beam-to-column connections, can be analysed by comparing the force-displacement curves and accumulated energy shown in Figs. 11 to 13.

Comparison between numerical and experimental results allows to conclude that the proposed stress-strain relationship is able to simulate, with a good level of accuracy until failure, the behaviour of a beam-to-column connection with top and seat web angle.

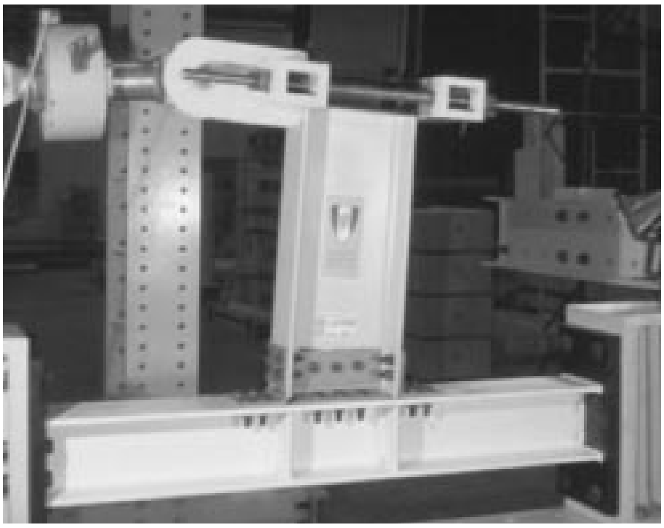

Figure 10. Beam-to-column connection studied

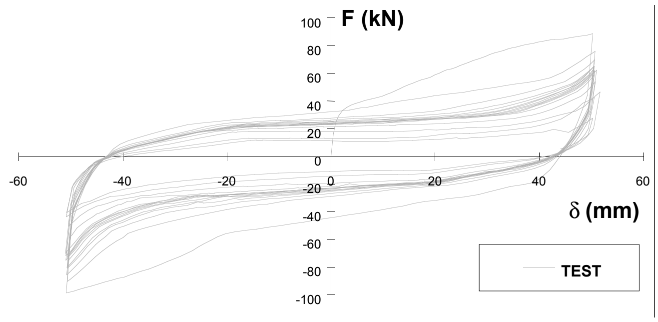

Figure 11. Experimental force – displacement curve

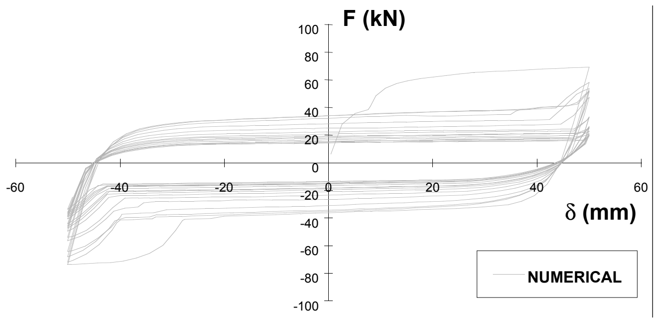

Figure 12. Numerical force – displacement curve

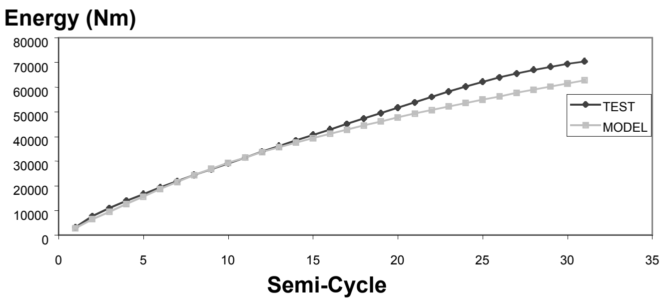

Figure 13. Comparison between numerical and experimental accumulated energy

The force-displacement curve, the accumulated energy, the Bauschinger effect and the damage accumulation are appropriately modelled. It should be pointed out that in this numerical simulation the input data are the geometric and mechanical characteristics of the connection, and the $\sigma-\varepsilon$ relationship of the steel.

7. CONCLUSIONS

A numerical model for the stress-strain relationship of steel, which takes into account complex loading conditions, the Bauschinger effect and the damage accumulation, is presented in this paper. Comparisons are drawn with uniaxial cyclic experiments. The proposed model generally predicts well the entire behaviour of the steel until failure. The Bauschinger effect, the shape of the curve, partial unloadings and the number of cycles until the failure are well simulated. Based on numerical simulation of the hysteretic behaviour of a beam-to-column connection, it is the authors' opinion that the proposed $\sigma-\varepsilon$ relationship can be easily implemented in structural analysis programs for the prediction of the inelastic behaviour of steel structures subjected to earthquake loads.

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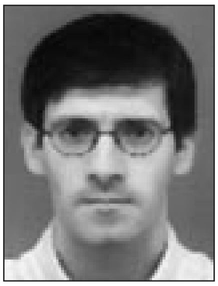

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